We note further that the expression $\square f_{nm}$ with account of the adopted notation (13.19) can be written in the form

$$\square f_{nm}=\square\psi_{nm}-\partial_n\partial^i\psi_{mi}-\partial_m\partial^i\psi_{ni}+\gamma_{nm}\partial^l\partial^i\psi_{li}.$$

This expression is also invariant under transformations (13.21) with any gauge vector a^n, but the operator $\square f_{nm}$ in this case will have the original form. We can simplify this operator if we note that in the gauge we have adopted the additional conditions (13.25) are satisfied. In this case we obtain

$$\square f_{nm}=\square\psi_{nm}. \tag{13.26}$$

We note that the operator $\square f_{nm}$ of (13.26) obtained is also invariant under gauge transformations (13.21) without violating the additional conditions (13.25). Relations (13.26) make it possible to rewrite the equations of the gravitational field in the form

$$\square f^{nm}=-16\pi I^{nm} \tag{13.27}$$

with the additional conditions

$$\partial_n f^{nm}=0. \tag{13.28}$$

It should be especially emphasized that the tensor current on the right side of Eq. (13.27) is concentrated only in matter.

We note also that the equations of the field theory of gravitation (13.27) can be formulated not only for inertial but also for noninertial coordinate systems, whereby in passing from one noninertial system to another the field equations are form-invariant for each infinite collection of noninertial coordinate systems. In the case of inertial coordinate systems, the field equations are Lorentz invariant in passing from one inertial system to another. This brings us to the necessity of extending the principle of relativity [4] which we formulate as follows: by no physical phenomena, including gravitational phenomena, can we determine whether we are at rest or in a state of uniform, progressive motion.

We emphasize that the relativity principle does not require constancy of the propagation speed of the front of an electromagnetic wave — the speed of light. It is natural that in the presence of interaction with external gravitational fields the speed of light is not constant.

14. Equations of Minimal Coupling

To close the theoretical scheme we must now indicate the coupling equation between the metric tensor of effective Riemannian space—time g_{ni} and the gravitational field f_{ni}.

Since the choice of coupling equation in the field theory of gravitaion is equivalent to the choice of a Lagrangian density for the interaction between the gravitational field and other fields of matter, we shall construct the coupling equation in a way analogous to the construction of an interaction Lagrangian in theories of other physical fields. Thus, for example, in electrodynamics a "minimal Lagrangian" is chosen as the density of the interaction Lagrangian.

Therefore, in the field theory of gravitation it is appropriate to choose as coupling equation an equation of minimal coupling which is the minimal required for the description of the present collection of experiments in the case of a weak gravitational field. In the linear approximation usually considered the tensor current I^{nm} of (13.22) must be taken in the absence of a gravitational field. Since in this approximation the sole physical symmetric tensor of second rank satisfying a conservation law is the energy—momentum tensor of matter, we require that the following correspondence be satisfied: in zeroth approximation in the gravitational field the tensor current I^{nm} must automatically go over into the energy—momentum tensor of matter:

$$I^{nm}(f_{li}=0)=T^{nm}. \tag{14.1}$$

This correspondence condition makes it possible to uniquely establish the structure of the coupling equation $g_{ni} = g_{ni}(f_{ml})$ in the linear approximation. Indeed, using the expressions (13.11), (13.18), and (13.22), we find that the correspondence condition (14.1) leads to the following coupling equation in linear approximation:

$$g_{ni}=\gamma_{ni}+f_{ni}-\frac{1}{2}\gamma_{ni}f. \tag{14.2}$$

It might be supposed that relation (14.2) is the equation of minimal coupling and is always satisfied not just in linear approximation of a weak field f_{ni}. A theory with such a coupling equation would then belong to the class of so-called "quasilinear" theories of gravitation (in the terminology of Will). However, as is shown in the work [24], any "quasilinear," asymptotically Lorentz-invariant theory of gravitation contradicts the results of experiments. Therefore, the relation (14.2) must only be the expansion of the minimal coupling equation up to linear terms in a weak field f_{ni}.

Thus, the equation of minimal coupling must be a quadratic equation in the field f_{ni}:

$$g_{ni}=\gamma_{ni}+f_{ni}-\frac{1}{2}\gamma_{ni}f+\frac{1}{4}[b_1 f_{nm}f_i^m+b_2 f_{ni}f+b_3\gamma_{ni}f_{ml}f^{ml}+b_4\gamma_{ni}f^2], \tag{14.3}$$

with parameters of minimal coupling b_1, b_2, b_3, and b_4 which are so far undetermined.

As we shall see below, the condition of coincidence of post-Newtonian expressions for the inertial and gravitational masses of a spherically symmetric body leads to the following relation between the parameters of minimal coupling: $2(b_1 + b_2 + b_3 + b_4) = 1$.

It would be possible to consider also more complex coupling equations which in the weak-field approximation go over into the minimal coupling equation (14.3). However, at present we have no justification for such complication, since the equation of minimal coupling (14.3) describes all gravitational experiments.

We therefore carry out all subsequent considerations on the basis of the equation of minimal coupling (14.3). Here we shall consider the condition of absence of singularities of the metric of the effective Riemannian space–time for finite values of the density of matter at the source of the gravitational field as the basic physical requirement imposing definite restrictions on the values of the parameters of minimal coupling. This assumption excludes the appearance in the field theory of gravitation of objects reminiscent of black holes.

Moreover, we require that there be no paradox of Olbers type in the description of the model of the universe.

It should be noted that because of the equation of minimal coupling (14.3) nondiagonal components of the metric tensor of Riemannian space–time g_{nm} can be nonzero even when the nondiagonal components of the gravitational field f_{nm} are equal to zero.

In order that the nondiagonal components of the tensor g_{nm} vanish when the corresponding nondiagonal components of the gravitational field are equal to zero, it is necessary and sufficient that $b_1 = 0$. In this case we arrive at the equation of simplest minimal coupling (the P–M coupling)

$$g_{nm}=\gamma_{nm}+f_{nm}-\frac{1}{2}\gamma_{nm}f_{li}+\frac{1}{4}[b_2 f_{nm}f+b_3\gamma_{nm}f_{li}+b_4\gamma_{nm}f^2] \tag{14.4}$$

The condition of coincidence of post-Newtonian expressions for the gravitational and inertial mass of a static, spherically symmetric body requires that the parameters of the P–M coupling (14.4) satisfy the relation $2(b_2 + b_3 + b_4) = 1$.

15. Conservation Laws in the Field Theory of Gravitation

Conservation laws valid for all theories of gravitation of class (A) were obtained in Sec. 12. The presence in theories of this class of a differential conservation law for the density of the total symmetric energy–momentum tensor of a system in flat space–time (12.18) makes it possible to obtain a corresponding integral conservation law.

In Cartesian coordinates we have

$$\partial_n[t_g^{ni}+t_M^{ni}]=0. \tag{15.1}$$

Integrating this expression over some volume V for i = 0 and assuming that across the surface bounding this volume there are no flows of matter, we obtain

$$-\frac{\partial}{\partial t}\int dV[t_g^{00}+t_M^{00}]=\oint dS_\alpha t_g^{0\alpha}. \tag{15.2}$$

Thus, in the radiation of gravitational waves the energy of the source must change, whereby if the gravitational waves carry positive energy the energy of the source must decrease.