

Electrical Engineering

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Notation

The library uses the symbol font for some of the notation and formulae. If the symbols for the letters ‘alpha beta delta’ do not appear here [α β δ] then the symbol font needs to be installed before all notation and formulae will be displayed correctly.

E	voltage source	[volts, V]	V	voltage drop	[volts, V]
G	conductance	[siemens, S]	X	reactance	[ohms, Ω]
I	current	[amps, A]	Y	admittance	[siemens, S]
R	resistance	[ohms, Ω]	Z	impedance	[ohms, Ω]
P	power	[watts]			

Ohm's Law

When an applied voltage **E** causes a current **I** to flow through an impedance **Z**, the value of the impedance **Z** is equal to the voltage **E** divided by the current **I**.

$$\text{Impedance} = \text{Voltage} / \text{Current} \qquad \mathbf{Z = E / I}$$

Similarly, when a voltage **E** is applied across an impedance **Z**, the resulting current **I** through the impedance is equal to the voltage **E** divided by the impedance **Z**.

$$\text{Current} = \text{Voltage} / \text{Impedance} \qquad \mathbf{I = E / Z}$$

Similarly, when a current **I** is passed through an impedance **Z**, the resulting voltage drop **V** across the impedance is equal to the current **I** multiplied by the impedance **Z**.

$$\text{Voltage} = \text{Current} * \text{Impedance} \qquad \mathbf{V = IZ}$$

Alternatively, using admittance Y which is the reciprocal of impedance Z:

$$\text{Voltage} = \text{Current} / \text{Admittance} \qquad \mathbf{V = I / Y}$$

Kirchhoff's Laws

Kirchhoff's Current Law

At any instant the sum of all the currents flowing into any circuit node is equal to the sum of all the currents flowing out of that node:

$$\mathbf{\Sigma I_{in} = \Sigma I_{out}}$$

Similarly, at any instant the algebraic sum of all the currents at any circuit node is zero:

$$\mathbf{\Sigma I = 0}$$

Kirchhoff's Voltage Law

At any instant the sum of all the voltage sources in any closed circuit is equal to the sum of all the voltage drops in that circuit:

$$\mathbf{\Sigma E = \Sigma IZ}$$

Similarly, at any instant the algebraic sum of all the voltages around any closed circuit is zero:

$$\mathbf{\Sigma E - \Sigma IZ = 0}$$

Thévenin's Theorem

Any linear voltage network which may be viewed from two terminals can be replaced by a voltage-source equivalent circuit comprising a single voltage source \mathbf{E} and a single series impedance \mathbf{Z} . The voltage \mathbf{E} is the open-circuit voltage between the two terminals and the impedance \mathbf{Z} is the impedance of the network viewed from the terminals with all voltage sources replaced by their internal impedances.

Norton's Theorem

Any linear current network which may be viewed from two terminals can be replaced by a current-source equivalent circuit comprising a single current source \mathbf{I} and a single shunt admittance \mathbf{Y} . The current \mathbf{I} is the short-circuit current between the two terminals and the admittance \mathbf{Y} is the admittance of the network viewed from the terminals with all current sources replaced by their internal admittances.

Thévenin and Norton Equivalence

The open circuit, short circuit and load conditions of the Thévenin model are:

$$\mathbf{V}_{oc} = \mathbf{E}$$

$$\mathbf{I}_{sc} = \mathbf{E} / \mathbf{Z}$$

$$\mathbf{V}_{load} = \mathbf{E} - \mathbf{I}_{load}\mathbf{Z}$$

$$\mathbf{I}_{load} = \mathbf{E} / (\mathbf{Z} + \mathbf{Z}_{load})$$

The open circuit, short circuit and load conditions of the Norton model are:

$$\mathbf{V}_{oc} = \mathbf{I} / \mathbf{Y}$$

$$\mathbf{I}_{sc} = \mathbf{I}$$

$$\mathbf{V}_{load} = \mathbf{I} / (\mathbf{Y} + \mathbf{Y}_{load})$$

$$\mathbf{I}_{load} = \mathbf{I} - \mathbf{V}_{load}\mathbf{Y}$$

Thévenin model from Norton model

Voltage = Current / Admittance

$$\mathbf{E} = \mathbf{I} / \mathbf{Y}$$

Impedance = 1 / Admittance

$$\mathbf{Z} = \mathbf{Y}^{-1}$$

Norton model from Thévenin model

Current = Voltage / Impedance

$$\mathbf{I} = \mathbf{E} / \mathbf{Z}$$

Admittance = 1 / Impedance

$$\mathbf{Y} = \mathbf{Z}^{-1}$$

When performing network reduction for a Thévenin or Norton model, note that:

- nodes with zero voltage difference may be short-circuited with no effect on the network current distribution,
 - branches carrying zero current may be open-circuited with no effect on the network voltage distribution.
-

Superposition Theorem

In a linear network with multiple voltage sources, the current in any branch is the sum of the currents which would flow in that branch due to each voltage source acting alone with all other voltage sources replaced by their internal impedances.

Reciprocity Theorem

If a voltage source \mathbf{E} acting in one branch of a network causes a current \mathbf{I} to flow in another branch of the network, then the same voltage source \mathbf{E} acting in the second branch would cause an identical current \mathbf{I} to flow in the first branch.

Compensation Theorem

If the impedance \mathbf{Z} of a branch in a network in which a current \mathbf{I} flows is changed by a finite amount $\delta\mathbf{Z}$, then the change in the currents in all other branches of the network may be calculated by inserting a voltage source of $-\mathbf{I}\delta\mathbf{Z}$ into that branch with all other voltage sources replaced by their internal impedances.

Millman's Theorem (Parallel Generator Theorem)

If any number of admittances $\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3, \dots$ meet at a common point P, and the voltages from another point N to the free ends of these admittances are $\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3, \dots$ then the voltage between points P and N is:

$$\mathbf{V}_{PN} = (\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 + \mathbf{E}_3\mathbf{Y}_3 + \dots) / (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \dots)$$

$$\mathbf{V}_{PN} = \Sigma\mathbf{E}\mathbf{Y} / \Sigma\mathbf{Y}$$

The short-circuit currents available between points P and N due to each of the voltages $\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3, \dots$ acting through the respective admittances $\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3, \dots$ are $\mathbf{E}_1\mathbf{Y}_1, \mathbf{E}_2\mathbf{Y}_2, \mathbf{E}_3\mathbf{Y}_3, \dots$ so the voltage between points P and N may be expressed as:

$$\mathbf{V}_{PN} = \Sigma\mathbf{I}_{sc} / \Sigma\mathbf{Y}$$

Joule's Law

When a current \mathbf{I} is passed through a resistance \mathbf{R} , the resulting power \mathbf{P} dissipated in the resistance is equal to the square of the current \mathbf{I} multiplied by the resistance \mathbf{R} :

$$\mathbf{P} = \mathbf{I}^2\mathbf{R}$$

By substitution using Ohm's Law for the corresponding voltage drop \mathbf{V} ($= \mathbf{I}\mathbf{R}$) across the resistance:

$$\mathbf{P} = \mathbf{V}^2 / \mathbf{R} = \mathbf{V}\mathbf{I} = \mathbf{I}^2\mathbf{R}$$

Maximum Power Transfer Theorem

When the impedance of a load connected to a power source is varied from open-circuit to short-circuit, the power absorbed by the load has a maximum value at a load impedance which is dependent on the impedance of the power source.

Note that power is zero for an open-circuit (zero current) and for a short-circuit (zero voltage).

Voltage Source

When a load resistance \mathbf{R}_T is connected to a voltage source \mathbf{E}_S with series resistance \mathbf{R}_S , maximum power transfer to the load occurs when \mathbf{R}_T is equal to \mathbf{R}_S .

Under maximum power transfer conditions, the load resistance \mathbf{R}_T , load voltage \mathbf{V}_T , load current \mathbf{I}_T and load power \mathbf{P}_T are:

$$\mathbf{R}_T = \mathbf{R}_S$$

$$\mathbf{V}_T = \mathbf{E}_S / 2$$

$$\mathbf{I}_T = \mathbf{V}_T / \mathbf{R}_T = \mathbf{E}_S / 2\mathbf{R}_S$$

$$\mathbf{P}_T = \mathbf{V}_T^2 / \mathbf{R}_T = \mathbf{E}_S^2 / 4\mathbf{R}_S$$

Current Source

When a load conductance \mathbf{G}_T is connected to a current source \mathbf{I}_S with shunt conductance \mathbf{G}_S , maximum power transfer to the load occurs when \mathbf{G}_T is equal to \mathbf{G}_S .

Under maximum power transfer conditions, the load conductance \mathbf{G}_T , load current \mathbf{I}_T , load voltage \mathbf{V}_T and load power \mathbf{P}_T are:

$$\mathbf{G}_T = \mathbf{G}_S$$

$$\mathbf{I}_T = \mathbf{I}_S / 2$$

$$\mathbf{V}_T = \mathbf{I}_T / \mathbf{G}_T = \mathbf{I}_S / 2\mathbf{G}_S$$

$$\mathbf{P}_T = \mathbf{I}_T^2 / \mathbf{G}_T = \mathbf{I}_S^2 / 4\mathbf{G}_S$$

Complex Impedances

When a load impedance \mathbf{Z}_T (comprising variable resistance \mathbf{R}_T and variable reactance \mathbf{X}_T) is connected to an alternating voltage source \mathbf{E}_S with series impedance \mathbf{Z}_S (comprising resistance \mathbf{R}_S and reactance \mathbf{X}_S), maximum power transfer to the load occurs when \mathbf{Z}_T is equal to \mathbf{Z}_S^* (the complex conjugate of \mathbf{Z}_S) such that \mathbf{R}_T and \mathbf{R}_S are equal and \mathbf{X}_T and \mathbf{X}_S are equal in magnitude but of opposite sign (one inductive and the other capacitive).

When a load impedance \mathbf{Z}_T (comprising variable resistance \mathbf{R}_T and constant reactance \mathbf{X}_T) is connected to an alternating voltage source \mathbf{E}_S with series impedance \mathbf{Z}_S (comprising resistance \mathbf{R}_S and reactance \mathbf{X}_S), maximum power transfer to the load occurs when \mathbf{R}_T is equal to the magnitude of the impedance comprising \mathbf{Z}_S in series with \mathbf{X}_T :

$$\mathbf{R}_T = |\mathbf{Z}_S + \mathbf{X}_T| = (\mathbf{R}_S^2 + (\mathbf{X}_S + \mathbf{X}_T)^2)^{1/2}$$

Note that if \mathbf{X}_T is zero, maximum power transfer occurs when \mathbf{R}_T is equal to the magnitude of \mathbf{Z}_S :

$$\mathbf{R}_T = |\mathbf{Z}_S| = (\mathbf{R}_S^2 + \mathbf{X}_S^2)^{1/2}$$

When a load impedance \mathbf{Z}_T with variable magnitude and constant phase angle (constant power factor) is connected to an alternating voltage source \mathbf{E}_S with series impedance \mathbf{Z}_S , maximum power transfer to the load occurs when the magnitude of \mathbf{Z}_T is equal to the

magnitude of Z_S :

$$(\mathbf{R}_T^2 + \mathbf{X}_T^2)^{1/2} = |\mathbf{Z}_T| = |\mathbf{Z}_S| = (\mathbf{R}_S^2 + \mathbf{X}_S^2)^{1/2}$$

Kennelly's Star-Delta Transformation

A star network of three impedances Z_{AN} , Z_{BN} and Z_{CN} connected together at common node N can be transformed into a delta network of three impedances Z_{AB} , Z_{BC} and Z_{CA} by the following equations:

$$Z_{AB} = Z_{AN} + Z_{BN} + (Z_{AN}Z_{BN} / Z_{CN}) = (Z_{AN}Z_{BN} + Z_{BN}Z_{CN} + Z_{CN}Z_{AN}) / Z_{CN}$$

$$Z_{BC} = Z_{BN} + Z_{CN} + (Z_{BN}Z_{CN} / Z_{AN}) = (Z_{AN}Z_{BN} + Z_{BN}Z_{CN} + Z_{CN}Z_{AN}) / Z_{AN}$$

$$Z_{CA} = Z_{CN} + Z_{AN} + (Z_{CN}Z_{AN} / Z_{BN}) = (Z_{AN}Z_{BN} + Z_{BN}Z_{CN} + Z_{CN}Z_{AN}) / Z_{BN}$$

Similarly, using admittances:

$$Y_{AB} = Y_{AN}Y_{BN} / (Y_{AN} + Y_{BN} + Y_{CN})$$

$$Y_{BC} = Y_{BN}Y_{CN} / (Y_{AN} + Y_{BN} + Y_{CN})$$

$$Y_{CA} = Y_{CN}Y_{AN} / (Y_{AN} + Y_{BN} + Y_{CN})$$

In general terms:

$$Z_{\text{delta}} = (\text{sum of } Z_{\text{star}} \text{ pair products}) / (\text{opposite } Z_{\text{star}})$$

$$Y_{\text{delta}} = (\text{adjacent } Y_{\text{star}} \text{ pair product}) / (\text{sum of } Y_{\text{star}})$$

Kennelly's Delta-Star Transformation

A delta network of three impedances Z_{AB} , Z_{BC} and Z_{CA} can be transformed into a star network of three impedances Z_{AN} , Z_{BN} and Z_{CN} connected together at common node N by the following equations:

$$Z_{AN} = Z_{CA}Z_{AB} / (Z_{AB} + Z_{BC} + Z_{CA})$$

$$Z_{BN} = Z_{AB}Z_{BC} / (Z_{AB} + Z_{BC} + Z_{CA})$$

$$Z_{CN} = Z_{BC}Z_{CA} / (Z_{AB} + Z_{BC} + Z_{CA})$$

Similarly, using admittances:

$$Y_{AN} = Y_{CA} + Y_{AB} + (Y_{CA}Y_{AB} / Y_{BC}) = (Y_{AB}Y_{BC} + Y_{BC}Y_{CA} + Y_{CA}Y_{AB}) / Y_{BC}$$

$$Y_{BN} = Y_{AB} + Y_{BC} + (Y_{AB}Y_{BC} / Y_{CA}) = (Y_{AB}Y_{BC} + Y_{BC}Y_{CA} + Y_{CA}Y_{AB}) / Y_{CA}$$

$$Y_{CN} = Y_{BC} + Y_{CA} + (Y_{BC}Y_{CA} / Y_{AB}) = (Y_{AB}Y_{BC} + Y_{BC}Y_{CA} + Y_{CA}Y_{AB}) / Y_{AB}$$

In general terms:

$$Z_{\text{star}} = (\text{adjacent } Z_{\text{delta}} \text{ pair product}) / (\text{sum of } Z_{\text{delta}})$$

$$Y_{\text{star}} = (\text{sum of } Y_{\text{delta}} \text{ pair products}) / (\text{opposite } Y_{\text{delta}})$$

Electrical Circuit Formulae

Notation

The library uses the symbol font for some of the notation and formulae. If the symbols for the letters 'alpha beta delta' do not appear here [α β δ] then the symbol font needs to be installed before all notation and formulae will be displayed correctly.

C	capacitance	[farads, F]	Q	charge	[coulombs, C]
E	voltage source	[volts, V]	q	instantaneous Q	[coulombs, C]

e	instantaneous E	[volts, V]	R	resistance	[ohms, Ω]
G	conductance	[siemens, S]	T	time constant	[seconds, s]
I	current	[amps, A]	t	instantaneous time	[seconds, s]
i	instantaneous I	[amps, A]	V	voltage drop	[volts, V]
k	coefficient	[number]	v	instantaneous V	[volts, V]
L	inductance	[henrys, H]	W	energy	[joules, J]
M	mutual inductance	[henrys, H]	Φ	magnetic flux	[webers, Wb]
N	number of turns	[number]	Ψ	magnetic linkage	[webers, Wb]
P	power	[watts, W]	ψ	instantaneous Ψ	[webers, Wb]

Resistance

The resistance R of a circuit is equal to the applied direct voltage E divided by the resulting steady current I :

$$R = E / I$$

Resistances in Series

When resistances R_1 , R_2 , R_3 , ... are connected in series, the total resistance R_S is:

$$R_S = R_1 + R_2 + R_3 + \dots$$

Voltage Division by Series Resistances

When a total voltage E_S is applied across series connected resistances R_1 and R_2 , the current I_S which flows through the series circuit is:

$$I_S = E_S / R_S = E_S / (R_1 + R_2)$$

The voltages V_1 and V_2 which appear across the respective resistances R_1 and R_2 are:

$$V_1 = I_S R_1 = E_S R_1 / R_S = E_S R_1 / (R_1 + R_2)$$

$$V_2 = I_S R_2 = E_S R_2 / R_S = E_S R_2 / (R_1 + R_2)$$

In general terms, for resistances R_1 , R_2 , R_3 , ... connected in series:

$$I_S = E_S / R_S = E_S / (R_1 + R_2 + R_3 + \dots)$$

$$V_n = I_S R_n = E_S R_n / R_S = E_S R_n / (R_1 + R_2 + R_3 + \dots)$$

Note that the highest voltage drop appears across the highest resistance.

Resistances in Parallel

When resistances R_1 , R_2 , R_3 , ... are connected in parallel, the total resistance R_P is:

$$1 / R_P = 1 / R_1 + 1 / R_2 + 1 / R_3 + \dots$$

Alternatively, when conductances G_1 , G_2 , G_3 , ... are connected in parallel, the total conductance G_P is:

$$G_P = G_1 + G_2 + G_3 + \dots$$

$$\text{where } G_n = 1 / R_n$$

For two resistances R_1 and R_2 connected in parallel, the total resistance R_P is:

$$R_P = R_1 R_2 / (R_1 + R_2)$$

$$R_P = \text{product} / \text{sum}$$

The resistance R_2 to be connected in parallel with resistance R_1 to give a total resistance R_P is:

$$R_2 = R_1 R_P / (R_1 - R_P)$$

$$R_2 = \text{product} / \text{difference}$$

Current Division by Parallel Resistances

When a total current I_P is passed through parallel connected resistances R_1 and R_2 , the voltage V_P which appears across the parallel circuit is:

$$V_P = I_P R_P = I_P R_1 R_2 / (R_1 + R_2)$$

The currents I_1 and I_2 which pass through the respective resistances R_1 and R_2 are:

$$I_1 = V_P / R_1 = I_P R_P / R_1 = I_P R_2 / (R_1 + R_2)$$

$$I_2 = V_P / R_2 = I_P R_P / R_2 = I_P R_1 / (R_1 + R_2)$$

In general terms, for resistances R_1, R_2, R_3, \dots (with conductances G_1, G_2, G_3, \dots) connected in parallel:

$$V_P = I_P R_P = I_P / G_P = I_P / (G_1 + G_2 + G_3 + \dots)$$

$$I_n = V_P / R_n = V_P G_n = I_P G_n / G_P = I_P G_n / (G_1 + G_2 + G_3 + \dots)$$

where $G_n = 1 / R_n$

Note that the highest current passes through the highest conductance (with the lowest resistance).

Capacitance

When a voltage is applied to a circuit containing capacitance, current flows to accumulate charge in the capacitance:

$$Q = \int i dt = CV$$

Alternatively, by differentiation with respect to time:

$$dq/dt = i = C dv/dt$$

Note that the rate of change of voltage has a polarity which opposes the flow of current.

The capacitance C of a circuit is equal to the charge divided by the voltage:

$$C = Q / V = \int i dt / V$$

Alternatively, the capacitance C of a circuit is equal to the charging current divided by the rate of change of voltage:

$$C = i / dv/dt = dq/dt / dv/dt = dq/dv$$

Capacitances in Series

When capacitances C_1, C_2, C_3, \dots are connected in series, the total capacitance C_S is:

$$1 / C_S = 1 / C_1 + 1 / C_2 + 1 / C_3 + \dots$$

For two capacitances C_1 and C_2 connected in series, the total capacitance C_S is:

$$C_S = C_1 C_2 / (C_1 + C_2)$$

C_S = product / sum

Voltage Division by Series Capacitances

When a total voltage E_S is applied to series connected capacitances C_1 and C_2 , the charge Q_S which accumulates in the series circuit is:

$$Q_S = \int i_S dt = E_S C_S = E_S C_1 C_2 / (C_1 + C_2)$$

The voltages V_1 and V_2 which appear across the respective capacitances C_1 and C_2 are:

$$V_1 = \int i_S dt / C_1 = E_S C_S / C_1 = E_S C_2 / (C_1 + C_2)$$

$$V_2 = \int i_S dt / C_2 = E_S C_S / C_2 = E_S C_1 / (C_1 + C_2)$$

In general terms, for capacitances C_1, C_2, C_3, \dots connected in series:

$$Q_S = \int i_S dt = E_S C_S = E_S / (1 / C_S) = E_S / (1 / C_1 + 1 / C_2 + 1 / C_3 + \dots)$$

$$V_n = \int i_S dt / C_n = E_S C_S / C_n = E_S / C_n (1 / C_S) = E_S / C_n (1 / C_1 + 1 / C_2 + 1 / C_3 + \dots)$$

Note that the highest voltage appears across the lowest capacitance.

Capacitances in Parallel

When capacitances C_1, C_2, C_3, \dots are connected in parallel, the total capacitance C_P is:

$$C_P = C_1 + C_2 + C_3 + \dots$$

Charge Division by Parallel Capacitances

When a voltage E_P is applied to parallel connected capacitances C_1 and C_2 , the charge Q_P which accumulates in the parallel circuit is:

$$Q_P = \int i_P dt = E_P C_P = E_P (C_1 + C_2)$$

The charges Q_1 and Q_2 which accumulate in the respective capacitances C_1 and C_2 are:

$$Q_1 = \int i_1 dt = E_P C_1 = Q_P C_1 / C_P = Q_P C_1 / (C_1 + C_2)$$

$$Q_2 = \int i_2 dt = E_P C_2 = Q_P C_2 / C_P = Q_P C_2 / (C_1 + C_2)$$

In general terms, for capacitances C_1, C_2, C_3, \dots connected in parallel:

$$Q_P = \int i_P dt = E_P C_P = E_P (C_1 + C_2 + C_3 + \dots)$$

$$Q_n = \int i_n dt = E_P C_n = Q_P C_n / C_P = Q_P C_n / (C_1 + C_2 + C_3 + \dots)$$

Note that the highest charge accumulates in the highest capacitance.

Inductance

When the current changes in a circuit containing inductance, the magnetic linkage changes and induces a voltage in the inductance:

$$d\psi/dt = e = L di/dt$$

Note that the induced voltage has a polarity which opposes the rate of change of current.

Alternatively, by integration with respect to time:

$$\Psi = \int e dt = LI$$

The inductance L of a circuit is equal to the induced voltage divided by the rate of change of current:

$$L = e / di/dt = d\psi/dt / di/dt = d\psi/di$$

Alternatively, the inductance L of a circuit is equal to the magnetic linkage divided by the current:

$$L = \Psi / I$$

Note that the magnetic linkage Ψ is equal to the product of the number of turns N and the magnetic flux Φ :

$$\Psi = N\Phi = LI$$

Mutual Inductance

The mutual inductance M of two coupled inductances L_1 and L_2 is equal to the mutually induced voltage in one inductance divided by the rate of change of current in the other inductance:

$$M = E_{2m} / (di_1/dt)$$

$$M = E_{1m} / (di_2/dt)$$

If the self induced voltages of the inductances L_1 and L_2 are respectively E_{1s} and E_{2s} for the same rates of change of the current that produced the mutually induced voltages E_{1m} and E_{2m} , then:

$$M = (E_{2m} / E_{1s})L_1$$

$$M = (E_{1m} / E_{2s})L_2$$

Combining these two equations:

$$M = (E_{1m}E_{2m} / E_{1s}E_{2s})^{1/2} (L_1L_2)^{1/2} = k_M(L_1L_2)^{1/2}$$

where k_M is the mutual coupling coefficient of the two inductances L_1 and L_2 .

If the coupling between the two inductances L_1 and L_2 is perfect, then the mutual inductance M is:

$$M = (L_1L_2)^{1/2}$$

Inductances in Series

When uncoupled inductances L_1 , L_2 , L_3 , ... are connected in series, the total inductance L_S is:

$$L_S = L_1 + L_2 + L_3 + \dots$$

When two coupled inductances L_1 and L_2 with mutual inductance M are connected in series, the total inductance L_S is:

$$L_S = L_1 + L_2 \pm 2M$$

The plus or minus sign indicates that the coupling is either additive or subtractive, depending on the connection polarity.

Inductances in Parallel

When uncoupled inductances L_1, L_2, L_3, \dots are connected in parallel, the total inductance L_P is:

$$1 / L_P = 1 / L_1 + 1 / L_2 + 1 / L_3 + \dots$$

Time Constants

Capacitance and resistance

The time constant of a capacitance C and a resistance R is equal to CR , and represents the time to change the voltage on the capacitance from zero to E at a constant charging current E/R (which produces a rate of change of voltage E/CR across the capacitance).

Similarly, the time constant CR represents the time to change the charge on the capacitance from zero to CE at a constant charging current E/R (which produces a rate of change of voltage E/CR across the capacitance).

If a voltage E is applied to a series circuit comprising a discharged capacitance C and a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_C across the capacitance and the charge q_C on the capacitance are:

$$\begin{aligned}i &= (E/R)e^{-t/CR} \\v_R &= iR = Ee^{-t/CR} \\v_C &= E - v_R = E(1 - e^{-t/CR}) \\q_C &= Cv_C = CE(1 - e^{-t/CR})\end{aligned}$$

If a capacitance C charged to voltage V is discharged through a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_C across the capacitance and the charge q_C on the capacitance are:

$$\begin{aligned}i &= (V/R)e^{-t/CR} \\v_R &= iR = Ve^{-t/CR} \\v_C &= v_R = Ve^{-t/CR} \\q_C &= Cv_C = CVe^{-t/CR}\end{aligned}$$

Inductance and resistance

The time constant of an inductance L and a resistance R is equal to L/R , and represents the time to change the current in the inductance from zero to E/R at a constant rate of change of current E/L (which produces an induced voltage E across the inductance).

If a voltage E is applied to a series circuit comprising an inductance L and a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_L across the inductance and the magnetic linkage ψ_L in the inductance are:

$$\begin{aligned}i &= (E/R)(1 - e^{-tR/L}) \\v_R &= iR = E(1 - e^{-tR/L}) \\v_L &= E - v_R = Ee^{-tR/L} \\\psi_L &= Li = (LE/R)(1 - e^{-tR/L})\end{aligned}$$

If an inductance L carrying a current I is discharged through a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_L across the inductance and the magnetic linkage ψ_L in the inductance are:

$$\begin{aligned}i &= Ie^{-tR/L} \\v_R &= iR = IR e^{-tR/L} \\v_L &= v_R = IR e^{-tR/L} \\\psi_L &= Li = LI e^{-tR/L}\end{aligned}$$

Rise Time and Fall Time

The rise time (or fall time) of a change is defined as the transition time between the 10%

and 90% levels of the total change, so for an exponential rise (or fall) of time constant \mathbf{T} , the rise time (or fall time) $\mathbf{t_{10-90}}$ is:

$$\mathbf{t_{10-90} = (\ln 0.9 - \ln 0.1)\mathbf{T} \approx 2.2\mathbf{T}}$$

The half time of a change is defined as the transition time between the initial and 50% levels of the total change, so for an exponential change of time constant \mathbf{T} , the half time $\mathbf{t_{50}}$ is :

$$\mathbf{t_{50} = (\ln 1.0 - \ln 0.5)\mathbf{T} \approx 0.69\mathbf{T}}$$

Note that for an exponential change of time constant \mathbf{T} :

- over time interval \mathbf{T} , a rise changes by a factor $\mathbf{1 - e^{-1}}$ (≈ 0.63) of the remaining change,
 - over time interval \mathbf{T} , a fall changes by a factor $\mathbf{e^{-1}}$ (≈ 0.37) of the remaining change,
 - after time interval $\mathbf{3T}$, less than 5% of the total change remains,
 - after time interval $\mathbf{5T}$, less than 1% of the total change remains.
-

Telecommunication



Copy of the original phone of Graham Bell at the *Musée des Arts et Métiers* in Paris

Telecommunication is the transmission of signals over a distance for the purpose of communication. In modern times, this process almost always involves the sending of electromagnetic waves by electronic transmitters but in earlier years it may have involved the use of smoke signals, drums or semaphore. Today, telecommunication is widespread and devices that assist the process such as the television, radio and telephone are common in many parts of the world. There is also a vast array of networks that connect these devices, including computer networks, public telephone networks, radio networks and television networks. Computer communication across the Internet, such as e-mail and instant messaging, is just one of many examples of telecommunication.

Telecommunication systems are generally designed by **telecommunication engineers**. Major contributors to the field of telecommunications include Alexander Bell who invented the telephone (as we know it), John Logie Baird who invented the mechanical television and Guglielmo Marconi who first demonstrated transatlantic radio communication. In recent times, optical fibre has radically improved the bandwidth available for intercontinental communication helping to facilitate a faster and richer Internet experience and digital television has eliminated effects such as snowy pictures and ghosting. Telecommunication remains an important part of the world economy and the telecommunication industry's revenue has been placed at just under 3% of the gross world product.

Key concepts

The basic elements of a telecommunication system are:

- a transmitter that takes information and converts it to a signal for transmission
- a transmission medium over which the signal is transmitted
- a receiver that receives and converts the signal back into usable information

For example, consider a radio broadcast. In this case the broadcast tower is the transmitter, the radio is the receiver and the transmission medium is free space. Often

telecommunication systems are two-way and devices act as both a transmitter and receiver or *transceiver*. For example, a mobile phone is a transceiver. Telecommunication over a phone line is called point-to-point communication because it is between one transmitter and one receiver, telecommunication through radio broadcasts is called broadcast communication because it is between one powerful transmitter and numerous receivers.

Signals can either be analogue or digital. In an analogue signal, the signal is varied continuously with respect to the information. In a digital signal, the information is encoded as a set of discrete values (e.g. 1's and 0's).

A collection of transmitters, receivers or transceivers that communicate with each other is known as a network. Digital networks may consist of one or more routers that route data to the correct user. An analogue network may consist of one or more switches that establish a connection between two or more users. For both types of network, a repeater may be necessary to amplify or recreate the signal when it is being transmitted over long distances. This is to combat attenuation that can render the signal indistinguishable from noise.

A channel is a division in a transmission medium so that it can be used to send multiple independent streams of data. For example, a radio station may broadcast at 96 MHz while another radio station may broadcast at 94.5 MHz. In this case the medium has been divided by frequency and each channel received a separate frequency to broadcast on. Alternatively one could allocate each channel a recurring segment of time over which to broadcast.

The shaping of a signal to convey information is known as modulation. Modulation is a key concept in telecommunications and is frequently used to impose the information of one signal on another. Modulation is used to represent a digital message as an analogue waveform. This is known as keying and several keying techniques exist — these include phase-shift keying, amplitude-shift keying and minimum-shift keying. Bluetooth, for example, uses phase-shift keying for exchanges between devices (see note).

However, more relevant to earlier discussion, modulation is also used to boost the frequency of analogue signals. This is because a raw signal is often not suitable for transmission over long distances of free space due to its low frequencies. Hence its information must be superimposed on a higher frequency signal (known as a carrier wave) before transmission. There are several different modulation schemes available to achieve this — some of the most basic being amplitude modulation and frequency modulation. An example of this process is a DJ's voice being superimposed on a 96 MHz carrier wave using frequency modulation (the voice would then be received on a radio as the channel "96 FM").

Society and telecommunication

Telecommunication is an important part of many modern societies. In 2006, estimates place the telecommunication industry's revenue at \$1.2 trillion or just under 3% of the gross world product. Good telecommunication infrastructure is widely acknowledged as important for economic success in the modern world both on a micro and macroeconomic scale. And, for this reason, there is increasing worry about the digital divide.

This stems from the fact that access to telecommunication systems is not equally shared amongst the world's population. A 2003 survey by the International Telecommunication Union (ITU) revealed that roughly one-third of countries have less than 1 mobile subscription for every 20 people and one-third of countries have less than 1 fixed line subscription for every 20 people. In terms of Internet access, roughly half of countries have less than 1 in 20 people with Internet access. From this information as well as educational data the ITU was able to compile a Digital Access Index that measures the overall ability of citizens to access and use information and communication technologies. Using this measure, countries such as Sweden, Denmark and Iceland receive the highest ranking while African countries such as Niger, Burkina Faso and Mali receive the lowest. Further discussion of the social impact of telecommunication is often considered part of communication theory.

History



A replica of one of Chappe's semaphore towers.

Early telecommunications

Early forms of telecommunication include smoke signals and drums. Drums were used by natives in Africa, New Guinea and South America whereas smoke signals were used

by natives in North America and China. Contrary to what one might think, these systems were often used to do more than merely announce the presence of a camp.

In 1792, a French engineer, Claude Chappe built the first fixed visual telegraphy (or semaphore) system between Lille and Paris. However semaphore as a communication system suffered from the need for skilled operators and expensive towers often at intervals of only ten to thirty kilometres (six to nineteen miles). As a result, the last commercial line was abandoned in 1880.

Telegraph and telephone

The first commercial electrical telegraph was constructed by Sir Charles Wheatstone and Sir William Fothergill Cooke and opened on 9 April 1839. Both Wheatstone and Cooke viewed their device as “an improvement to the [existing] electromagnetic telegraph” not as a new device.

On the other side of the Atlantic Ocean, Samuel Morse independently developed a version of the electrical telegraph that he unsuccessfully demonstrated on 2 September 1837. Soon after he was joined by Alfred Vail who developed the register — a telegraph terminal that integrated a logging device for recording messages to paper tape. This was demonstrated successfully on 6 January 1838. The first transatlantic telegraph cable was successfully completed on 27 July 1866, allowing transatlantic telecommunication for the first time.

The conventional telephone was invented by Alexander Bell in 1876. Although in 1849 Antonio Meucci invented a device that allowed the electrical transmission of voice over a line. Meucci’s device depended upon the electrophonic effect and was of little practical value because it required users to place the receiver in their mouth to “hear” what was being said. The first commercial telephone services were set-up in 1878 and 1879 on both sides of the Atlantic in the cities of New Haven and London.

Radio and television

In 1832, James Lindsay gave a classroom demonstration of wireless telegraphy to his students. By 1854 he was able to demonstrate a transmission across the Firth of Tay from Dundee to Woodhaven, a distance of two miles, using water as the transmission medium. In December 1901, Guglielmo Marconi established wireless communication between Britain and the United States earning him the Nobel Prize in physics in 1909 (which he shared with Karl Braun).

On March 25, 1925, John Logie Baird was able to demonstrate the transmission of moving pictures at the London department store Selfridges. Baird’s device relied upon