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# Fuzzy Measurement of Sustainability

NOVA

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# PREFACE

People from many disciplines talk about sustainable development (SD) without having a concrete definition or idea what it is. Politicians and decision-makers, biologists and environmentalists, engineers and scientists, philosophers and sociologists often mean different things when they mention SD. To take an extreme example, a neoclassical economist might define SD as continual economic growth without regard to the environmental situation, whereas at the other end of the spectrum, a deep ecologist might define it as the preservation and restoration of the ecosystem, ignoring the effect on the economy completely.

It has become clear throughout the last few decades leading up to the time this book was written that our society is facing environmental problems of global magnitude such as species extinction, population explosion, global warming, water shortages, exhaustion of fisheries, and deforestation, among others, which have economic and social repercussions. Many people doubt that the world we shall bequeath to our descendants will be all healthy, supportive, and beautiful as the one we inherited from our parents. The fundamental questions then of the SD discussion are: Can we continue our present course without destroying part or most of our environment and society? And if we have to change course (which is currently the consensus opinion), how do we do it?

This book addresses one of the most fundamental questions of SD: How do we define and measure in the mathematical sense sustainability? In effect it summarizes our research in this field for the last 10 years. The mathematical model which we use to evaluate sustainability serves both as a definition and as a measuring scheme. This is done with the help of fuzzy logic, which is well suited to perform reasoning in fields where concrete mathematical models do not exist and concepts such as sustainability are multifaceted and often subjective. Fuzzy reasoning, in some respects, emulates human thinking and relies on expert opinion and knowledge as well as subjective evaluations.

This multistage model is developed step-by-step, in the end providing sustainability assessments of various aspects of a society on a scale from zero to one, such as the environmental or societal situation of a country. All nations of the world for which reliable data exist are then ranked according to their sustainability as measured by the model. A sensitivity analysis reveals the most important components of sustainability for each country.

The book is intended for undergraduate students, graduate students, and practitioners working in the field of sustainability. It is based on a course Yannis Phillis has taught at the University of California at Los Angeles (UCLA) and the Technical University of Crete (TUC) at the senior/graduate level. The mathematical knowledge needed to follow the book is minimal and is all contained in the text. Computational guidance is provided for those who would like to write their own model.

The book is organized as follows:

Chapter 1 discusses sustainability, its problems and meanings. Chapter 2 provides an introduction to fuzzy logic which is brief but sufficient for the needs of the book. Chapter 3 introduces certain indicators of sustainability. Chapter 4 is the main part of the book, which describes the fuzzy model in detail and provides results. Chapter 5 extends the concepts of previous chapters to the measurement of the sustainability of organizations such as corporations. This last subject of corporate sustainability is very much in fashion nowadays: many major companies as part of their social responsibility publish annual reports that show their contribution to the sustainability of the region where they operate.

We are indebted to several people who have patiently contributed their knowledge and work in this field. We thank our former PhD students Lucas Andrianos and Victor Kouloumpis who worked very hard for several years with us to make the model functional. Professor Nikos Tsourveloudis assisted us with Chapter 2. Nili Phillis edited the English text. Ben Davis of UCLA edited the text and contributed data for the corporate sustainability model.

Chania, June 2008

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*Chapter 1*

# INTRODUCTION

## 1.1. WHO IS INTERESTED IN SUSTAINABILITY?

Something is wrong with the way we humans treat our earth. An environmental crisis is unfolding before our very eyes in a period of just a few decades. Such time spans are equal nearly to zero when compared to the 4.5 billion years planet earth has been in existence.

Lately it is common for environmental problems to figure prominently in the media. We have become accustomed to daily stories about unusual weather phenomena attributed to global warming, toxic spills, depletion of fisheries, deforestation, species extinction, desertification, air, soil, and water pollution and so on. Most importantly, these problems have achieved their prominence due to an ever increasing volume of scientific research, which, in most cases, has performed a commendable job in exposing the facets of the environmental predicament.

The present human population of 6.7 billion is expected to reach almost 8 billion by 2025 (PRB 2008). The environmental impact of an additional 1.3 billion humans is not known precisely, but it is certain that it will worsen the pressure on the ecosystems and their dwindling resources such as water, fisheries, species, etc. The strain of overpopulation is not only environmental. Many societal aspects such as health, economy, education and the political system itself will be tried.

We live in a fast changing world with ever increasing uncertainty about what is in store in the future. Dramatic changes occur in only a few years or just decades and so the time the society is given to adjust to these changes is very short. In the past, climate change progressed mostly over thousands of years, allowing humans and other species enough time to evolve or move to more suitable habitats. In the present global warming era, species have only 50 to 100 years to find new hospitable places. But some plant and animal species move slowly or don’t have where to go and these are doomed.

A question thus arises as to the consequences of our actions to us and the environment: How sustainable is our present course? Such a question begs a definition and a reliable scheme for assessing sustainability. This is the central subject of this book.

Sustainability is a concept discussed nowadays by politicians, environmentalists, biologists, sociologists, economists, engineers, philosophers, and ordinary citizens. This happens because sustainability concerns all humans and can be used by many disciplines, scientific or other.

Politicians love catch phrases such as sustainable development. There is even talk in some countries of changing the ministries of economy into ministries of sustainable development (at least the latter sounds more attractive to the voters). Politics play a central role in the discussion of sustainable development. After all, it is economic activity coupled with political decisions that affect the environmental or the social system. The European Union (EU), for example, has decided that its member countries should cut down their fossil fuel use in an effort to reduce greenhouse gases emissions. To achieve this end, a concerted effort is made to install large numbers of wind turbines in all EU member countries: this process involves primarily political decisions.

It is quite natural for environmentalists to be involved in the sustainability debate since the environment already is under enormous strain. The understanding is now settling in that without a robust environment we cannot have a healthy economy over a long period. The economy relies on such things as clean air, water, soil, minerals, and other resources, which are products of the physical and biological environment.

Biologists measure populations of animals and plants, find correlations, statistics and relationships between and within species, and provide scientific explanations of such phenomena as exploding or shrinking populations. They develop indicators of sustainable fish or plant harvesting, indicators of biodiversity, etc. They also explain the inner workings of species and ecosystems which then lead to explanations of human environmental impact, albeit often a-posteriori.

No society exists without an economy nowadays, excluding perhaps tiny groups of hunter-gatherers in the Amazon or Africa. An economy ordinarily requires labor, capital, matter, and energy to function. We know from the laws of physics that matter and energy can neither be created nor destroyed but they may be transformed into other forms of matter and energy or into each other, although the transformation of matter into energy is not of interest here.

An economy then takes this useful matter and energy and, with the aid of technology, transforms it into finished products. These products, after their useful life, become garbage or are partially recycled at the expense of energy: in either case, the products generate pollution. Take the example of a car, which is made of iron, copper, nickel, lead, plastics and other materials. Mining of these materials results in pollution. The mined matter is then transferred to a factory where a purified form of the metals is extracted using energy. Next, the auto maker combines all the materials to produce the car. Each and every stage so far has polluted the environment. Finally the user of the car, after some time, takes the car to a dump where it is partially recycled and partially left to rot. Once again the end product is pollution taken up by the environment. If this pollution is qualitatively and quantitatively within the absorbing capacity of the environment, the problem ends there. If it's not, however, we end up with contamination in the form of unwanted substances in the air, water, and soil.

In any case, as shown in Figure 1.1, the economic activity, with the aid of technology, inevitably generates pollution. Of course, this is not done just for the fun of it: the economic process is a sine qua non condition for the existence of currently 6.7 billion people. Otherwise we would go back to a few hundreds of thousands of vulnerable and primitive hunter-gatherers. The tool of this economic process is technology, the child of science, and its end product is utility and pollution.

Technology changes all the time. It provides tools for the detection and measurement of pollutants as well as systems that clean up the environment when possible. The aim of new technologies is to produce goods with minimal adverse environmental impact. It therefore becomes clear that both economists and engineers have a stake in the sustainability debate.

Based on ideology, economists generate ideas about growth and development which influence both society and the environment. Sustainable development is an idea that means different things to capitalists, ecological economists or Marxists. Sociologists, on the other hand, ask questions about the importance of sustainability to society and the mechanism of raising awareness among people. Equally important is the question of finding motivations to mobilize people in the direction of improving sustainability.

Sustainability looks to the future and in some way guarantees an acceptable social and ecological system for future generations. Questions about the rights of as yet unborn people belong to the realm of philosophy as do questions about the rights of other species.

Finally, we the common citizens are the ones who enjoy highly polluting sports utility vehicles, electronic gadgets and toys, and air conditioned houses. We consume, often irrationally, we pollute and thus, we breathe dirty air, drink water laced with toxic substances, and eat unhealthy food. We also vote, enjoy life and, when bad things happen, we suffer. Without doubt we have the greatest stakes in the sustainability case.

We all could develop sustainably provided:

1) We develop a definition or description of sustainable development
2) We possess mechanisms and we have the political will to do so.

This book deals with the first condition and provides some mathematical tools to answer it. The second question, although of the utmost importance and complexity, is outside the scope of this book. The main reason is that question 2 is not scientific but has to do with values, ethics, and politics.

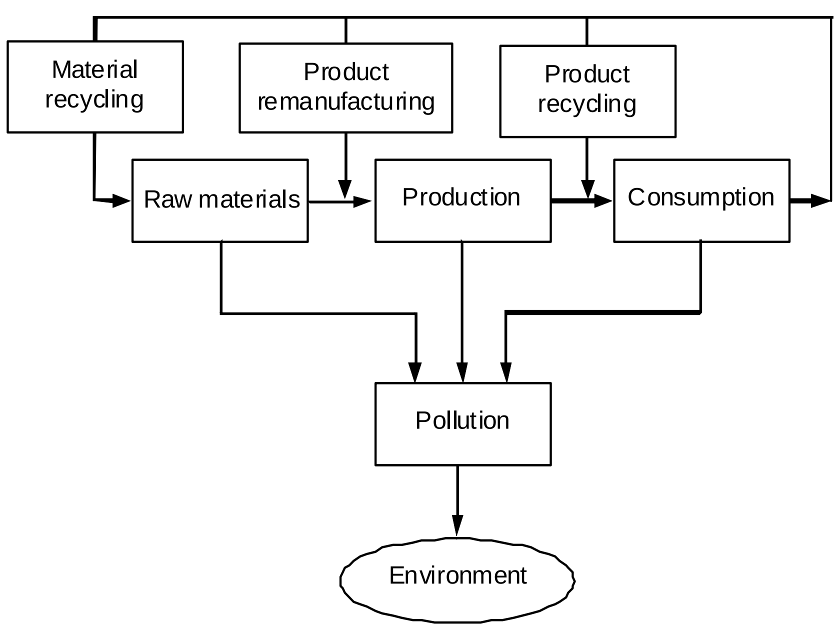


Figure 1.1. The economic process.

## 1.2. DIFFICULTIES OF ACHIEVING SUSTAINABILITY

People are speaking about sustainability more now because the environmental crisis has become obvious to most of them. A society is an extremely complex system and the possibilities of action within this system many. When a problem arises, its solutions depend on the knowledge, values, and the willingness of the decision makers and the citizens to undertake a certain course of action. Thus complex societal problems such as sustainability are not only technical but primarily cultural and political (Prugh *et al.* 2000).

Roughly speaking, a society is sustainable if it provides basic necessities and happiness to the present generation and generations to come over a very long time. We shall come back to this.

In the past, sustainability was not a major issue if at all, because the size of human population and the level of consumption, on average, were such that the terrestrial or aquatic ecosystems were not threatened by human actions on a global scale. The scales of economies were small and there was always a frontier where humans could expand.

Today, in contrast, we have reached or exceeded the capacity of the earth to supply resources and absorb pollution. Anything we do has environmental consequences which, when multiplied by all people and then added up, explain for example why global warming is occurring. In the past, human society relied on the consumption of solar energy to satisfy its needs for food and cover. Plants and animals are the products of this solar energy through photosynthesis and they provide food, building materials, clothing, medicines, a stable climate, a stable atmosphere, esthetic pleasure and so on. Fossil fuels are also products of this solar energy: plants and microorganisms trapped in the strata of the earth provide coal, oil, and natural gas, which in essence represent the nuclear energy of the sun which these species received via photosynthesis thousands of years ago.

Today we consume the ecosystem itself to support our growth. We remove forests and fish beyond replenishment, pump water beyond recharging, drive species to extinction, poison the air, water, and soil, sometimes irreversibly.

To see our impact on the environment numerically, let us present two examples:

First, suppose you live in Greece around the time this book was written.

- You have a GNI PPP (gross national income in purchasing power parity, which is gross national income converted to international dollars—international dollars correspond to goods and services one could buy in the U.S. with a certain amount of money) of U.S. $32,520 (PRB 2008).
- You are expected to live on average 77 years if you are male or 81 years if you are female (nature also loves women).
- You wake up in the morning in a room with central heating in winter or air conditioning in summer, you turn on a few lights, make your coffee, toast your bread and then you drive or ride a bus or train to work. All in all by the end of the day your consumption has led directly or indirectly to the emission of 26.4 kilograms (kg) of carbon dioxide ($CO_2$) or equivalently 7.2 kg of carbon (C). In a year you have burdened the environment with over 2.6 tons of C which partly contributes to global warming (International Atomic Energy Agency 2003).

- If you wear a gold wedding ring you have contributed 3 tons of discharge at a mine in South Africa or the U.S. (47% of gold is recycled, thus polluting less). Make this 10 to 20 tons of discharge if you wear a gold watch (Gardner and Sampat 1998).
- For lunch you eat a 250 gram (g) beef steak. If this meat was produced in Brazil, it destroyed 5 $m^2$ of tropical forest, making a small contribution to global warming. If the animal was one year old, it had produced 27 kg of methane ($CH_4$), a very potent greenhouse gas. To obtain a further picture add 750 liters (lt) of water and 1.7 kg of grain to make the 250 g steak (Durning and Brough 1991). If you top the steak with 100 g of rice, then your lunch dish required about 90 g of nitrogen fertilizer, of which 80 g escaped directly into the environment contributing to global warming, eutrophication, water and air pollution.
- Your car requires 160 tons of water for its steel parts and tires. Its battery contains about 8 kg of lead which has generated about 310 kg of pollution at a mine in Australia or the U.S. (73% is recycled). The car has about 10 kg of copper which has generated 990 kg of discharge somewhere in Chile or the U.S. (60% is recycled) (Gardner and Sampat 1998).
- For dinner you decide to eat a 300 g farmed trout and a lettuce salad. You then contribute to the release of 400 g of particles, mostly organic, in a river in Epirus (Northern Greece), 19 g of ammonia, 4 g of nitrites and nitrates, 5 g of phosphorus, and small quantities of antibiotics, formaldehyde and other disinfectants. This fish consumed 700 g of grain whereas your lettuce was grown with nitrogen and pesticide inputs (Papoutsoglou 1996).
- By the end of the day you have generated 0.8 kg of garbage (50% organic, 20% paper) which usually ends up in some dump, often illegal (Eurostat 2005).

Now suppose you live in the U.S.

- You have a GNI PPP of U.S. $45,850.
- You are expected to live on average 75 years if you are male, 2 years less than a Greek (perhaps the latter enjoys life more), or 81 years if you are female.
- You have a 55% chance to be overweight or obese.
- You consume 200 g or 53 teaspoons of sweeteners per day, mostly through processed foods.
- You generate 5.5 tons of C per year or 15 kg per day, twice that of a Greek.
- Your car costs more environmentally because you use it more: In the U.S., 3 kg of C releases per day are due to transportation.
- Every day you use materials such as newspapers, paper and plastic bags, cars, appliances, gravel, sand, nitrogen, copper, zinc, etc. All in all, excluding fuel and food, you have consumed 101 kg of materials or 36,850 kg per year, or 2,838 tons per lifetime. Every day you generate 2 kg of garbage, 1/3 of which is packaging (Gardner and Sampat 1998).

It has been estimated (Power 1996) that 90% of the economic activity in rich countries satisfies or aims to satisfy wants, which later become needs. Such is the story of cable TV, cellular phones, fax machines, SUV’s, electronic gadgets, large houses, and air travel. In some sense we are all trapped in an endless cycle of consumption. Still, consumption patterns differ, despite a tendency to homogenization aided by the enormous advances in communication, transportation, and international trade. People exhibit different consumption behavior in the U.S., Greece, Nepal, Chad, or Peru; this is due to their different economic power and in a great measure to their personal and social values. Thus sustainability, while dealing with the economic activity, is also about values. The environmental behavior of a nation is also affected by values among other things. The environmental ethic in Northern Europe for example, is quite advanced, although one may argue that the Northern Europeans enriched themselves by destroying enough of their environment so they can afford now to protect whatever remains of it. Others argue that poor countries supply rich countries with cheap labor, resources, and dumps for their toxic or radioactive garbage, becoming heavily polluted in the process. However, the point that sustainability is mainly a matter of values remains valid. There are many nongovernmental organizations, scientists, or citizens who raise awareness about unfair treatment of poor countries and press for legislation and effective measures against this type of exploitation.

In all western type democracies, a small group of politicians, advisers, and lobbyists influence or directly make all the decisions of the state, whereas the rest of us are just "consumers." The great ancient Athenian leader Pericles in his famous speech "Epitaph," honoring the dead Athenians of the Peloponnesian War, called anyone who does not participate in politics "useless." Of course personal participation in public affairs is possible when population is small, just a few hundred of thousands. Even in Ancient Athens, however, women were excluded from the public gatherings where debates took place and decisions were made. In the end a few thousand people were deciding for about 300,000 but this is a far cry from the present "democratic" practice where often a handful decide for hundreds of millions.

Sustainability is about values, opinions, arguments, consensus or in one word politics which, of course, needs to be based on good science to be correct and beneficial. There are many examples in history, and even today, of more or less directly democratic systems besides Athens. Such systems might play a very important role in the process of transforming us from consumers into informed citizens who strive for sustainability. The reader may find interesting ideas about the subject of politics and sustainability in Prugh *et al.* (2000).

## 1.3. WHAT IS SUSTAINABILITY?

The biological and physical environment, in two words the global ecosystem, provides the economy with:

a) Resources such as wood, metals, minerals, fuels, food, drugs, water, air, fiber and so on.
b) Services as for example the cycles of $H_2O$, C, $CO_2$, N, $O_2$; photosynthesis; soil formation.
c) Mechanisms to absorb waste.

According to an attempt to monetize all three global services (Costanza *et al.* 1997), their total value in 1997 was $33 trillion/year. This number could be disputed on various grounds, but its enormity gives us an idea of the importance of the environment.

Economic growth is based on these three services and since the global ecosystem does not grow, economic growth cannot continue indefinitely. If this is so, and since the human mind cannot stop inventing new things and improving existing systems and processes, what should be done to preserve a basic level of satisfaction of necessities that would provide happiness to our society for a long time? In other words, how can we achieve sustainability?

It is claimed by several economists that substituting one form of capital for another solves the problem of finiteness of the ecosystem and the concomitant existence of limits. A worker can be replaced by a robot, wood for the shipping industry by iron, or metals for aircraft by synthetic materials. But substitution has its limits too, which result from ecological limits; in the long run, total output will go down. Replacing harpoon whaling by modern ships depletes the population of whales and substituting traditional nets with driftnets depletes fisheries. More importantly, substitution often is impossible owing to the scale of the global ecosystem or the laws of nature. There is no substitute for a stable climate which we are changing rapidly with dire consequences as well as there are no substitutes for extinct species and their services to nature.

One of the most basic economic indicators of the state of an economy is the gross national product or GNP, which is the total value of all final goods and services produced in a country in one year, plus income earned by its citizens abroad, minus income earned by foreigners in the country. The total value of final goods and services produced within a country is the gross domestic product (GDP). GNP is a useful indicator, but also one-sided and deceptive; if someone cuts down all forests or depletes fisheries, the GNP goes up even though economic catastrophy is near. Such growth of the GNP is unsustainable. In fact, GNP growth and consumerism go hand in hand.

There have been attempts to correct the single-sidedness of GNP or GDP. One such attempt (Venetoulis and Cobb 2004) defines the Genuine Progress Indicator (GPI). GPI includes the value of household work, parenting, volunteer work, services of customer durables, highways and streets, which GDP ignores. It also subtracts defensive expenses such as auto accidents, social costs such as cost of crime, and depreciation of environmental assets.

As an example, the GDP and GPI per capita for the U.S, in the year 2000 were $35,000 and $11,554 respectively.

Sustainable development does not necessarily mean growth, but improvement of the various societal sectors, as for example health, education, or the state of the environment. So what is sustainable development (SD)? To some people it is a contradiction in terms. There are economists who believe that SD means:

- Sustaining economic growth indefinitely

while others see it as:

- The ability to maintain desired social values, institutions, cultures or other social characteristics

or as:

- Development (improvement) that can be continued for a long time.

In 1980, the International Union for the Conservation of Nature and Natural Resources (IUCN) wrote in its World Conservation Strategy (WCS) (Lélé 1991) that it espouses "the overall aim of achieving sustainable development through the conservation of living resources." Since then the concept of SD gained momentum as the environmental degradation intensified and became obvious to many. Today most proponents of SD take it to mean:

- The existence of environmental and economic conditions needed to sustain human well-being at a given level over a very long time.

In more general terms, SD according to the Brundtland Report (WCED 1987) is:

- Development that meets the needs of the present without compromising the ability of future generations to meet their own needs.

In a slightly different way, IUCN defined SD as:

- Development that improves the quality of human life within the carrying capacity of supporting ecosystems.

SD was the central subject of the Earth Summit held in Rio de Janeiro in 1992. Agenda 21, which resulted from the meeting, gives a comprehensive list of actions needed to achieve SD. Leaders from over 150 states committed themselves to undertaking those actions that will render development sustainable.

It is common to classify SD as strong and weak:

- *Strong sustainability* is primarily focused on the environment and ignores economics such as the cost of achieving sustainability. It considers pollution, emissions, biodiversity, soil erosion etc.
- *Weak sustainability* is basically economic sustainability. It ignores environmental inputs to the economy and considers consumption, economic growth, financial value etc.

In this book sustainability integrates both aspects, environmental and economic. Two questions arise in the context of SD:

1. What is the space over which sustainability is considered?

   *Answer*. Ecosystem, a region, a country, and since pollution often has no borders, the globe. As the scale becomes smaller, the boundaries of the system which we examine become more uncertain. The sustainability of California, for example, depends on regional attributes such as local pollution, water resources, coastal areas, agriculture, education, health etc. but also external factors, e.g., climate, ozone depletion, water resources of the Colorado river, foreign imports and so on. In this book the most common scale of SD is that of a country.

2. What is the time horizon of sustainability?

   *Answer*. It depends on the specific attribute. For example, the climate exhibits several periodicities due to a change of the earth's orbit from almost circular to elliptic with a periodicity of 100,000 years. Also the axis of motion of the earth has two periodicities of

40,000 and 20,000 years respectively. Thus, the climate naturally oscillates between cold and warm with a variation in average global temperature of 5 °C or more. Incidentally, such climatic changes occur over periods of thousands of years. Today's global warming occurs within only a few decades.

The fluidity of time scales in the context of SD is shown in two more examples: pest problems have time scales of the order of 20 years. The life expectancy of species on the other hand is between 1 and 5 million years on average, although from the fossil record we know of species that survived only a few thousand years before becoming extinct.

Perhaps the most important aspect of time is that of uncertainty. We simply cannot plan our society for thousands of years into the future. Often even a few decades are beyond our predictive and planning capabilities. We have only long-term goals and intentions and all the time we make adaptive decisions which bring us as close to our goals as possible.

In the sequel a mathematical methodology will be developed whereby sustainability will be assessed via fuzzy logic. In effect the model that assesses sustainability serves also as a definition of the concept.

## 1.4. PROBLEMS-QUESTIONS

**1.1.** Give an example where substitution is impossible. Give one more where substitution leads to output reduction.

**1.2.** Outline two changes of national accounting that would make GDP more representative of the sustainability of a country.

**1.3.** List five values that improve sustainability. Provide brief explanations.

**1.4.** Energy consumption is the main contributor to $CO_2$ emissions. Energy is used in the following areas:

- transportation
- households
- industries – agriculture
- food production
- recreation
- tourism

Outline 10 actions that would make these sectors more sustainable.

**1.5.** Atmospheric concentrations of $CO_2$ in ppm (parts per million) in volume followed an almost linear trajectory between 1985 and 2000. The corresponding concentrations were

345.9 ppm and 369.5 ppm. If this trend continues what will the corresponding concentrations be in 2050 and 2100?

*Chapter 2*

# INTRODUCTION TO FUZZY LOGIC

*Fuzzy logic* was introduced by Zadeh (1965) as an extension of the classical two-valued logic, in which a proposition is either true or false and an object either belongs or does not belong to a set. Zadeh studied the concept of vagueness by assuming that propositions and set memberships are true with degrees ranging from 0 (100% false) to 1 (100% true). This method can handle incomplete knowledge and inexact or vague data in a systematic way.

In this book we develop fuzzy models to assess the sustainability of nations, geographical regions, and organizations.

## 2.1. WHY ASSESS SUSTAINABILITY VIA FUZZY LOGIC?

Sustainability is a multifaceted concept for which there is no widely accepted definition or measurement method. To reveal its dimensions and acquire a better understanding of the concept, we analyze it by following a top-down approach. As discussed in the previous chapter, sustainability has two broad dimensions, ecological and human. In the next chapters we shall explain how these two components break down into a number of secondary components and variables. In defining the sustainability of nations, for example, water availability and water quality are considered to be basic components of the ecological sustainability. Water sustainability is in turn assessed from data about annual availability of renewable water resources, water withdrawals, releases of water pollutants, availability of wastewater treatment plants, and so on. Such variables are used as basic inputs in sustainability calculations.

Therefore, *mathematically, sustainability is a composition of functions of several variables which, in turn, are also composite functions of more primitive variables*. There are two reasons why it is not possible to determine these functions explicitly:

1) Sustainability is an inherently vague and complex concept and cannot be described, let alone measured, by traditional mathematics. Policy makers and scientists often prefer natural language expressions rather than equations or numerical values in assessing sustainability. For example, degrees of law enforcement, the state of civil liberties, and the state of human rights in a country, which are important components of human sustainability, are often obtained by subjective assessments.