

FUNDAMENTALS OF RAIL VEHICLE DYNAMICS

GUIDANCE AND STABILITY

A.H. WICKENS

Loughborough University, UK

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Arrangement of the book

Sections are numbered serially within each chapter. If reference is made to a section within the chapter containing the section, the section number is cited as a single number. Otherwise, a section is identified by two numbers separated by a decimal point, the first number referring to the chapter in which the section appears, and the second identifying the section within the chapter.

Equations are numbered serially within each section. If reference is made to an equation within the section containing the equation, the equation number is cited as a single number. If reference is made elsewhere in the same chapter then the equation number is cited as a two-figure number and if reference is made in another chapter all three numbers—chapter, section and equation are cited.

Figures and tables are numbered by chapter.

Preface

The fundamental method of guidance of the railway vehicle is the coned and flanged wheelset. Whilst facilitating guidance in curves, coning can give rise to sustained lateral oscillations, termed hunting. This oscillation induces forces which can cause damage to both vehicle and track and there can be, at least, discomfort to the passenger and, at worst, the risk of derailment. Inadequate steering on curves can have similar consequences. This book concentrates on the resulting problem of the conflict between guidance and stability and its resolution by proper design of the suspension connecting the wheels and car body of the railway vehicle.

The invention of the wheelset, the progressive development of the bogie and the various schemes of articulation which have been developed over the years in order to resolve the design conflict between stability and steering, all predate the theory of railway vehicle dynamics. Engineering insight brought railway technology a long way but empirical methods were not adequate once the railway renaissance started and train speeds increased. A fundamental change in railway technology took place in which the empirical evolution of railway bogies was replaced by a more scientific and numerate approach. This approach has been very successful; for example, not only has stable operation of steel wheel on steel rail vehicles been demonstrated at speeds of over 500 km/h (more than double the speed of the fastest train fifty years ago) but the analytical and predictive capabilities now available have stimulated a rising tide of innovative designs.

The detailed modelling of the dynamics of railway vehicles is made possible by the several excellent computer packages that are available, which provide sufficiently detailed and validated mathematical models that can be used with confidence in engineering design and development. These models permit the simulation of the actual motion on a specified stretch of track so that the performance of a specific design can be analysed, or a particular incident recreated. Thus, by simulation the overall performance of a vehicle can be checked. Realism is, of course, essential in design but equates to complexity, and computer output must be tempered with understanding and scepticism. It is important, therefore, that fundamental principles are well understood.

This book is concerned with the fundamental principles of guidance and stability, which are a consequence of the mechanics of wheel-rail interaction as embodied in the equations of motion. For research purposes, where the objective is to achieve an understanding of an innovative system or a particular problem, simple models can be very useful and can provide productive insights. Analytical studies which describe the mechanics of various phenomena by the simplest model possible can be used to explore new suspension and vehicle design concepts.

Attention will be concentrated on the configuration and parametric design of the bogie, in relation to steering, dynamic response and stability. Therefore the treatment of the various configurations of vehicle do not simply concentrate on a current typical set of parameters but attempt to consider the consequences of the complete range of parameters open to the designer. By this approach, it is possible to see why much of current practice, though it pre-dates the availability of theory, is the way it is. Moreover, it becomes clear why many innovations failed in the past.

Because an important consequence of a more analytical approach is to separate out the dynamic properties of a system from the detailed design of its components the latter will not be discussed. Moreover, the application of active controls to steering and ride control (including body tilting) will not be covered. Active systems will play a large part in the future and those working in the field will require a sound grounding in passive systems. As the emphasis is on ride quality and guidance, a frequency range of roughly 0 – 15 Hz is of prime interest. This makes it possible to consider that, in general, wheelsets and track (except in the areas of contact) are rigid and that car bodies are without flexibility. This means that some significant phenomena are not discussed here. Moreover, simple forms of suspension elements are assumed. The more straightforward problems of response in the vertical plane, or in the longitudinal direction are not addressed.

The basic concepts are described in Chapter 1. A detailed discussion of the equations of motion follows in Chapter 2 in which a compromise has been made between the mathematical rigour of some investigators and the ad-hoc use of Newton's Laws of others. In this Chapter, though an engineering approach has been followed, great reliance has been placed on the careful derivations of Professor de Pater. The following Chapters deal with the single wheelset and then with progressively more complex configurations of vehicle. If possible, simple analytical results have been derived as these, if available, provide the best basis for understanding the mechanics of the systems involved. All numerical results have been obtained using standard commercially available software for numerical computation.

It is a pleasure to acknowledge the stimulus and help I have received over the years from colleagues, too numerous to mention here, at British Rail Research, Loughborough University and through the International Association of Vehicle System Dynamics.

A. H. Wickens
Idridgehay, 2002

1

Basic Concepts

1.1 Introduction

The railway train running along a track is one of the most complex dynamical systems in engineering. It has many degrees of freedom, the interaction between wheel and rail involves both complex geometry of wheel tread and rail head and non-conservative forces generated by relative motion in the contact area, and there are many non-linearities.

The long history of railway engineering provides many practical examples of dynamical problems which have degraded performance and safety. The two essential features of operation, running in a train of vehicles and guidance by the track, cause problems which are unique to railways. Inadequate guidance on curves results in high lateral forces between wheel and rail, rapid wear of wheels and rails and the possibility of derailment. Dynamic and static instabilities, and excessive response to track irregularities and other features of track geometry, can result in poor ride quality and high stresses and can contribute to derailment. Operation in a train involves the control of forces acting between the vehicles in the train as the propulsive and braking forces are varied in response to the train traversing hills and valleys. High frequency interaction between wheel and rail can lead to damage to the contacting surfaces and corrugation of the rails, and excessive noise and vibration.

The dynamics of the railway vehicle represents a balance between the forces acting between the wheel and the rail, the inertia forces and the forces exerted by the suspension and articulation. Of these, the basic characteristics of the wheel-rail interface such as friction, geometry, and the elasticity in the contact area are hardly under the control of the designer. But the configuration, suspension and forms of articulation can be varied over a wide range of possibilities, limited mainly by the degree of complexity considered acceptable for each application. The objective of suspension design is, therefore, to control the motion of the railway vehicle so that good ride quality is achieved, at the same time dynamic loads and the tendency to derail are reduced to acceptable levels, whilst running on track with geometry that is economically acceptable.

In a complete model of the dynamics of a railway vehicle, the vehicle is considered to be assembled from wheelsets, car bodies and intermediate structures which are flexible, and which are connected by components such as springs and dampers. Similarly, the vehicle is considered to run on a track which has a complex structure

with elastic and dissipative properties. Each major component has six rigid body degrees of freedom plus additional degrees of freedom representing the elastic distortion of the component. In the latter case, these additional degrees of freedom might represent a finite element model of the structure or a series of natural modes of vibration. The track can be modelled as a continuous structure with a moving interface at the points of contact, where the interaction between wheel and rail is dependent on the relative motion. This kind of model, with varying assumptions, is provided by various computer software packages which are used in the engineering design and analysis of railway vehicles. So, one objective of the study of the dynamics of railway vehicles is the development of sufficiently detailed and validated mathematical models that permit the simulation of the actual motion, on a specified stretch of line, so that the performance of a specific design can be analysed, or a particular incident recreated. Thus, by simulation, the overall performance of an existing or projected vehicle can be checked and design decisions made.

A second objective of the study of railway vehicle dynamics is to develop analytical or numerical models describing the mechanics of various phenomena by the simplest model possible. These can be used to explore new suspension and vehicle concepts and to develop a basis for physical understanding and insight. Ideally, not only analysis but synthesis is required in which various possibilities for design are exposed. Simpler models are typically generated by simplifying assumptions and in this book, concerned with guidance and stability, these are that

- the vehicle has a longitudinal plane of symmetry (parallel to the direction of motion on straight track) making it possible, under certain conditions, to separate equations governing those motions which are symmetric with respect to the plane of symmetry from those which govern anti-symmetric motions;
- variations in longitudinal motion are not considered so that the vehicle moves at constant forward speed;
- the motions of interest are at low frequencies and, in most cases, flexibility of components can be neglected.

It is the objective of this chapter to explain the basic concepts of stability and guidance of railway vehicles as a preliminary to more detailed mathematical analysis.

1.2 The Railway Wheelset

The basic unit of a railway vehicle is the wheelset, Figure 1.1. The conventional wheelset of today has the following features: it consists of two wheels fixed on a common axle, so that each wheel rotates with a common angular velocity and a constant distance between the two wheels is maintained. Flanges are provided on the inside edge of the treads and the flange-way clearance allows, typically, ± 7 – 10 mm of lateral displacement to occur before flange contact. Whilst many wheelsets commence life with purely coned treads, typically coned at $1/20$ or $1/40$, these treads wear rapidly in service, so that the treads come to possess curvature in the transverse direction. Similarly, rails also possess curvature in the transverse direction. All these

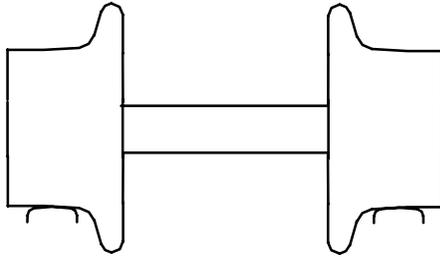


Figure 1.1 Railway wheelset.

features contribute to the behaviour of the railway vehicle as a dynamic system, and it is important to consider their purpose.

The conventional railway wheelset has a long history [1] and seems to have evolved by a process of trial and error. Naturally, in the pioneering days of the early railways most attention was concentrated on reducing rolling resistance so that the useful load that could be hauled by horses could be multiplied. Another major problem was the lack of strength and resistance to wear of the materials then available. Moreover, the level of adhesion between rolling wheel and the track was unknown. As a result, many possibilities were tried. An obvious step was to fit wheels with cylindrical treads. However, if the wheels are fixed on the axle and the treads are intended to be cylindrical very slight errors in parallelism would induce large lateral displacements which would be limited by flange contact. There is no guidance until flange contact and thus a wheelset with cylindrical treads tends to run in continuous flange contact. The position of the flange, either inside or outside the rails, was controversial well into the nineteenth century. Nor was there agreement as to whether the wheels should be rigidly fixed to an axle or free to revolve on the axle, though the usual practice seemed to be that wheels were fixed to the axle. The play allowed between wheel flange and rail was initially minimal. In the early 1830s the flange-way clearance was opened up with the objective of reducing the lateral forces between wheel and rail.

A further important point is that the geometry of the wheel and rail as it has evolved is particularly favourable for the method of switching which involves a minimum of moving parts and only small gaps in the running surfaces of the rails.

It is not known when coning of the wheel tread was first introduced. It would be natural to provide a smooth curve uniting the flange with the wheel tread, and wear of the tread would contribute to this. Moreover, once wheels were made of cast iron, taper was normal foundry practice. The purpose of coning was partly to reduce the rubbing of the flange on the rail, and partly also to ease the motion of the vehicle in curves.

A wheelset with coned wheels in a curve can maintain a pure rolling motion if it moves outward and adopts a radial position. Redtenbacher [2] provided the first theoretical analysis in 1855 which is illustrated in Figure 1.2. From the geometry in this figure it can be seen that there is a simple geometric relationship between the

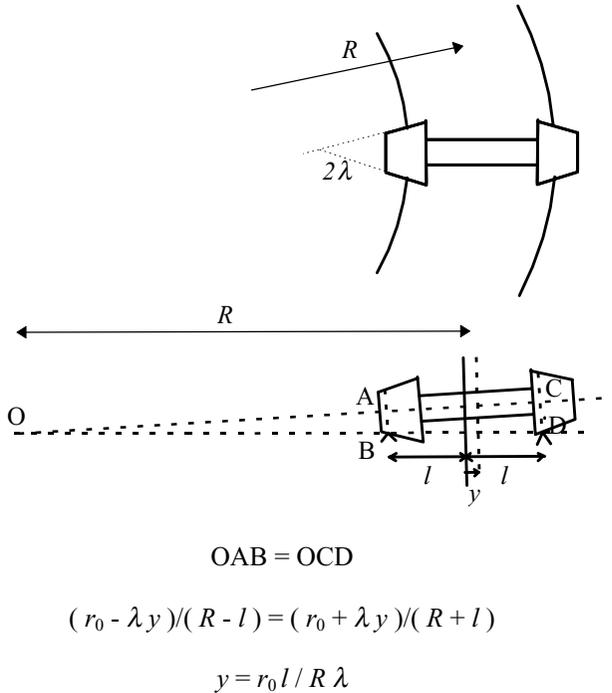


Figure 1.2 Redtenbacher's formula for the rolling of a coned wheelset on a curve.

lateral movement of the wheelset on a curve y , the radius of the curve R , the wheel radius r_0 , the lateral distance between the points of contact of the wheels with the rails $2l$ and the conicity λ of the wheels in order to sustain pure rolling. In practice a wheelset can only roll round moderate curves without flange contact, and a more realistic consideration of curving requires the analysis of the forces acting between the vehicle and the track.

It can be seen, in broad terms, why the wheelset adopted its present form. If the flange is on the inside the conicity is positive and as the flange approaches the rail there will be a strong steering action tending to return the wheelset to the centre of the track. If the flange is on the outside the conicity is negative and the wheelset will simply run into the flange and remain in contact as the wheelset moves along the track. Another factor is the behaviour in sharp curves. If the flange is on the inside then the lateral force applied by the rail to the leading wheelset is applied to the outer wheel and will be combined with an enhanced vertical load. As explained later, this diminishes the risk of derailment. With outside flanges the lateral force applied by the rail applied to the inner wheel which has a reduced vertical load and thus the risk of derailment is increased. These factors can be easily demonstrated with the aid of model wheelsets [3].

Thus, it can be seen that for small displacements from the centre of straight or slightly curved track the primary mode of guidance is conicity and it is on sharper

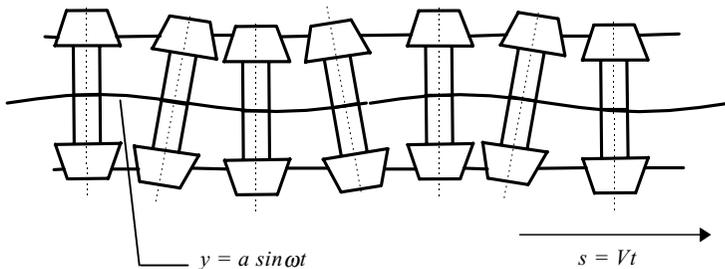
curves and switches and crossings that the flanges become the essential mode of guidance. Though this appears to be a modern view, in 1838 Brunel [4] wrote

The flanges are a necessary precaution but they ought never to touch the rail and therefore they cannot be said to keep the wheels on the rails. They ought not to come into action except to meet an accidental, lateral force. A railway with considerable curves might be travelled over with carriages at any velocity and with wheels without flanges. The wheels are made conical, the smaller circumference at the outer edge. The pair of wheels are fixed to the axle and thus if anything throws the wheels in the slightest degree to one side the wheel is immediately rolling on a larger circumference than the other and the tendency to roll back is introduced. The carriage is kept always in the middle of the track. A beautiful arrangement.

As a concept, this view led to many significant improvements in the design of rail-vehicle suspensions in the 20th century.

Coning of the wheel tread was well established by 1821. George Stephenson in his *Observations on Edge and Tram Railways* [5] stated that

It must be understood the form of edge railway wheels are conical that is the outer is rather less than the inner diameter about 3/16 of an inch. Then from a small irregularity of the railway the wheels may be thrown a little to the right or a little to the left, when the former happens the right wheel will expose a larger and the left one a smaller diameter to the bearing surface of the rail which will cause the latter to loose ground of the former but at the same time in moving forward it gradually exposes a greater diameter to the rail while the right one on the contrary is gradually exposing a lesser which will cause it to loose ground of the left one but will regain it on its progress as has been described alternately gaining and loosing ground of each other which will cause the wheels to proceed in an oscillatory but easy motion on the rails.



$$\frac{1}{R} = - \frac{d^2 y}{ds^2} = \frac{\omega^2 y}{V^2} = \frac{\omega^2 r_0 l}{V^2 R \lambda}$$

Figure 1.3 Derivation of Klingel's formula for the kinematic oscillation of a wheelset from Redtenbacher's formula in Figure 1.2.

This is a very clear description of what is now called the kinematic oscillation, as shown in Figure 1.3.

Thus, if a wheelset is rolling along the track and is displaced slightly to one side, the wheel on one side is running on a larger radius and the wheel on the other side is running on a smaller radius. Because the wheels are mounted on a common axle one wheel will move forward faster than the other because its instantaneous rolling radius is larger. Hence, if pure rolling is maintained, the wheelset moves back into the centre of the track – a steering action is provided by the coning. However, the wheelset overshoots the centre of the track and the result is the kinematic oscillation.

In 1883 Klingel gave the first mathematical analysis of the kinematic oscillation [6] and derived the relationship between the wavelength Λ and the wheelset conicity λ , wheel radius r_0 and lateral distance between the contact points between wheels and rails $2l$ as

$$\Lambda = 2\pi (r_0 l / \lambda)^{1/2}$$

This simple formula follows purely from the geometry of Figure 1.3, and is consistent with Redtenbacher's formula for the wheelset in a curve. Since distance along the track $s = Vt$ where V is the forward speed and t is time, Klingel's formula shows that, as the speed is increased, so will the frequency of the kinematic oscillation. Very little else can be deduced about the dynamical behaviour of railway vehicles which must come from a consideration of the forces acting.

1.3 Creep

Pure rolling rarely takes place, and wheels and rails are not rigid. The normal load between wheel and rail causes local elastic deformation and an area of contact, the contact patch, is formed. In the case where the surfaces of the wheels and rails are smooth and have constant curvature in the vicinity of the contact patch, Hertz [7] showed that the contact patch was elliptical in shape, and the distribution of normal pressure between wheel and rail over the contact patch is semi-ellipsoidal.

If a longitudinal force is applied to the wheel, so that it is braked, a deviation from the pure rolling motion occurs. The deviation in relative velocity divided by the forward speed of the wheel is referred to as the longitudinal creepage. Similarly, lateral creepage is defined as the (incremental) relative lateral velocity divided by the forward speed. In addition, relative angular motion between wheel and rail about the normal to the contact patch is referred to as spin. If the longitudinal creepage is small, it is accommodated by elastic strains in the vicinity of the contact patch. As the wheel rotates, unstrained material enters the contact patch at its leading edge. As the material moves through the contact patch, the relative velocity between the wheel and rail equals the rate of change of strain so that the surfaces are locked together. The magnitude of the resulting longitudinal tangential stress increases linearly with distance from the leading edge. Similarly, lateral creepage gives rise to lateral tangential stresses. Both longitudinal and lateral creepage therefore generate forces which are directly proportional to the corresponding creepage. When there is spin, the pattern of elastic strain is more complicated. In this case, as the material moves

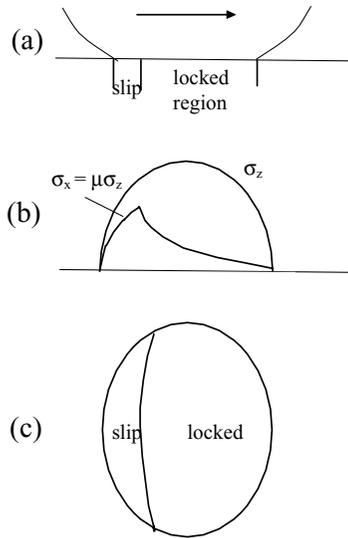


Figure 1.4 The contact patch between wheel and rail (a) elevation showing locked region of adhesion at leading edge and region of slip at trailing edge (b) normal pressure σ_z and tangential traction applied by wheel to rail σ_x (c) contact patch in plan view.

through the contact region the relative velocity between wheel and rail is directly proportional to the distance from the centre of the contact region and therefore the strain field becomes curved. As a consequence, a lateral force is generated (the couple about the common normal is small and may be safely neglected).

As the creepages and spin increase, the tangential stresses increase, and where these stresses exceed the normal pressure multiplied by the coefficient of friction, slipping takes place. The result is that the area of adhesion at the front of the contact patch in which the surfaces are locked together progressively reduces as the creepage increases, Figure 1.4. The relationship between the creep force and creepage is then as shown in Figure 1.5(a). For sufficiently large creepage, slipping takes place over the whole contact patch and the creep force is equal to the normal force multiplied by the coefficient of friction.

If both longitudinal and lateral creep occur simultaneously then for small creepages the creep forces can be superposed, but for larger creepages in the area of slipping the tangential stresses act in a direction opposite to the local resultant relative velocity. The result is that then all the creep forces are influenced by both lateral and longitudinal and lateral creepages and spin. Though the lateral force is proportional to spin for small values of the spin, for large values of the spin slipping takes place over a large part of the contact patch and the lateral force reduces to zero. The relationship between lateral force and spin is therefore as shown in Figure 1.5(b).

It was Carter's [8] introduction of the creep mechanism into the theory of lateral dynamics that was the crucial step in developing a realistic model of the wheelset.